Abstract- Chaos reveals a nonlinear behavior of chaotic system with the extreme sensitivity to initial conditions. Chaos adequately justify the complicated behavior of phenomena by presenting a very simple models comprises a limited equations which involve the deterministic, dynamical systems in continuous time space to present the copacetic approximation of real world. This is of particular relevance in the control of chaos as it is so complicated in regard of its nonlinearity behavior. In chaotic system solutions never converge to a specific numbers and vary chaotically from one amount to the other next. A tiny perturbation in a chaotic system may result in chaotic, periodic, or stationary behavior. Modern controllers are introduced for controlling the chaotic behavior. In this research an adaptive Fuzzy Logic Controller (AFLC) is proposed to control the Duffing’s chaotic system. This is an adaptive progressed fashion provide the empowered control facility in facing of nonlinear systems. AFLC methodology is an advanced control fashion yielding to both robustness and smooth motion in chaotic system control.

Keywords- AFLC; FLC; Membership Function

I. INTRODUCTION

Chaos exhibits the solutions for certain nonlinear equations in which they specify the inordinate sensitivity to initial conditions. The variables can dramatically differ with changes in the initial conditions and don’t converge to a particular numbers but varies chaotically from one value to another’s. Chaos manifests its applicability accidentally through Edward Lorenz’s research on weather prediction in1961. Lorenz simplifies the weather equations to the ordinary differential ones and reveals that the solutions are quite different with slight changes in the initial conditions. An early proponent of chaos theory was discovered by Henri Poincaré in the 1880s in his deep perusal in the three-body problem.

This is so that the cause and effect are not proportional no way. Any small variation in the initial conditions has been yield to the vast diverging in the outcomes. This feature is well known as the butterfly effect. System behavior is described by the initial conditions concerning with no random elements. This behavior comprises the deterministic system which is fully non predictable in nature and is introduced as deterministic chaos that is well known as chaos.

In accordance to the chaos theory the vast majority of phenomena that previously assumed with the sophisticated understanding might be adequately described by the simple models. These have advance applications in engineering, biology, philosophy and economics. In engineering, designers desire to control the chaotic system to achieve the stability using geometric approach, adaptive control and optimal solution. In this research an AFLC is developed for controlling the chaos in a defined trajectory.

II. DUFFING’S CHAOTIC SYSTEM

As a mathematical view, the chaos behaves like non-linear equations, with the significant characteristic in emerging in extreme sensitivity of solutions to their initial conditions. It is surprising that a simplest equations exhibit complex chaotic behaviors. Chaos could be occurred in systems of autonomous ordinary differential equations ODEs with as few as three variables and two quadratic nonlinearities, which was first propounded by Lorenz in 1963. The chaos system with two equilibrium points has been shown as second-order autonomous differential equation (1):

\[ X' = X' \]
\[ X'' = -X - 0.1X + \cos(\omega t) \]  

(1)

It depicts the sinusoidal driven mass with a nonlinear spring using the cubic restoring force (\(-x^3\)) and linear damping (\(-1x\)) that is a special case of Duffing’s oscillator (2):

\[ X'' + bX' + k_1X + k_2X^3 - A\sin(\omega t) = 0 \]  

(2)

In which \(b = 0.1\) (damping ratio); \(k_1 = 0; k_2 = 1; A = 1\) and.

This model could be assign for any symmetric oscillator like a mass on a spring driven with a sufficiently huge amplitude results the nonlinear restoring force.

The Duffing’s oscillator represents the actual model for the various phenomena like a magnetoelastic buckled beam or a nonlinear electronic circuit. (3).
\[
d\frac{d^2x}{dt^2} + k \frac{dx}{dt} + f(x) = e(t)
\]

(3)

Where \(e(t)\) is a function with a periodic of \((2\pi)\). So the \(\cos(2t)\) in (1) should be considered as \(\cos(2\pi/2)\).

\[X_2 = -X_1 - 1 \times X_1 + \cos t\]

(4)

This is a simple and deterministic equation but in reality the certain random phenomena with a very tiny attribute is observed which is uncertain factors between causes and effect that is usually neglected in mathematical model of physical system. In fact these systems almost include nonlinearity in nature that causes they behaves as a chaotic system.

### III. CHAOS CONTROL

Chaos control depicts in stabilization category by Stimulating using small system perturbations enforcing system to track one of its unstable periodic orbits. Only tiny perturbation is considered in order to avoidance of excessive variations in the dynamic of the system. Several techniques have been offered in chaotic control, which have been developed into three categories: the first one relay on non-feedback control that emphasizes on open loop control through the periodic system excitation (a disturbance-based technique) was first proposed by Lima and Pettini. So the stabilization is occurred towards a periodic state. The other is Pyragas method based on time-delayed feedback in continuous control. And the last one is OGY (Ott, Grebogi and Yorke) method or the Poincaré map linearization. The last two methods complete the algorithm of control by determining the previous orbits of the chaotic system. OGY method applies the disturbances through the feedback control to adjust the accessible parameters so that the chaotic system stabilizes via one of the unstable periodic orbits which are existed in the system. This disturbance of control will be applied at the time that orbit of chaotic system crosses a given Poincaré section in a way that the chaotic trajectory is near to the stable plate of the unstable periodic orbits. A major impediment in the OGY method is evident in case of the rapid systems while the response of the current computer-aided system are limited. On the other hand the applied noise can be yield in occasional bursts if the trajectory moves far away from the controlled periodic orbit. In the Pyragas method; a continuous linear feedback updated in each iteration. As the prior knowledge of the periodic orbit is not necessary in this technique and as the parameters have not varied in a fast time and the feedback term contains a delayed corresponds to the period of the unstable periodic orbits so this fashion can be apply for a rapid systems. A drawback of this method is the limitation in range of parameters means that the vast changes in parameters ultimate to eventually unstable chaotic system. Radu-Emil Precup, Marius L. Tomescu, Stefan Preitl suggest the Fuzzy Controllers for Takagi-Sugeno FLC meant for stabilizing the Lorenz system. O. Calvo, J. H. E. Cartwright proposed FLC for controlling the chaos. Both needs to well-studied chaotic system for adjusting membership functions. In this research adaptive fuzzy logic controller is investigated for controlling Duffing’s chaotic system including two equilibrium points. Scientific research has shown that fuzzy logic systems can be used to approximate any nonlinear function defined on a compact set. Adaptive fuzzy logic controllers evolve the Fuzzy logic concept contributing with the adaption law to form one of the best fashions in the word of control for applying in chaotic systems Fig. 1. Fuzzy Logic Controller (FLC) is reported as a non-analytic alternative for controlling complex applications. Fuzzy logic theory was introduced by Lotfi A. Zadeh in University of California “Berkeley” in 1965. FLC applies the fuzzy logic concept to inset the fuzzy algorithms based on linguistically rules.

![Adaptive Fuzzy Logic concept](image1)

**FIGURE 1: Adaptive Fuzzy Logic concept**

Using AFLC designers are released to determine the precise mathematical model of system and satisfy the vast adaption that is vital for a rapid variation which may be caused in the dynamic of nonlinear system. AFLC comprises the basic fuzzy logic concept equipping with an adaption learning algorithm.

![AFLC Main topology](image2)

**FIGURE 2: AFLC Main topology**

Adaption low adjusts the parameters of FLC through the training process so that the FLC rules automatically updated among the control process. Rules and system parameters are generated through the AFLC and human knowledge and expert information is downright only in the initialization.
stage. So if the knowledge was not assuring the dynamic of system it could be changed through the adaption procedure of parameters values, even in the undertrained conditions.

For a multi Input system with a unity output, the inputs variable could be shown as $x_j$, $j = 1, 2, \ldots, n$. Input parameters are assign as $(i_1, i_2, \ldots, i_n)$; $i_1 = 1, 2, \ldots, N_1$; $i_2 = 1, 2, \ldots, N_2$; $i_n = 1, 2, \ldots, N_n$. Inputs membership functions are defined as $A_i^j$. $C_{i_1, i_2, \ldots, i_n}$ are to set the output membership functions. $R_{i_1, i_2, \ldots, i_n}$ are the fuzzy implication:

$$R_{i_1, i_2, \ldots, i_n}(x, y) = A_i^j(x_1) \cdot A_i^j(x_2) \cdots A_i^j(x_n) \cdot C_{i_1, i_2, \ldots, i_n}(y) \quad (6)$$

Where “$*$” indicates T norm, $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{U}$ and $y \in \mathbb{V}$.

$$U = \sum_{i_1, i_2, \ldots, i_n} \varepsilon_{i_1, i_2, \ldots, i_n} (x, y) \quad (7)$$

The adjustable parameters $\zeta_{i_1, i_2, \ldots, i_n}$ are defined as $\Theta$. Control action and adaption low are defined as:

$$U = \Theta^T \tilde{x}(x) \quad (8)$$

$$\Theta^0 = \gamma \Theta^T P_{e, 0} \xi(x) \quad (9)$$

Lyapunov function is defined by (10) to satisfy the system stability during the control procedure:

$$V = \frac{1}{2} \varepsilon^T P e + b(\theta^* - \Theta)^T (\theta^* - \Theta) / 2 \gamma \quad (10)$$

The membership functions could be chosen as Gaussian form:

$$A_i^j(x_j, P_j, q_i) = \exp \left( -\frac{(x_j - P_j)^2}{2q_i^2} \right) \quad (11)$$

Six membership functions are defined as follows for both $x_j$ and $x_n$ chaotic states:

$$\mu_{P_0}^j(x(j)) = \exp(-x_j(1.5j)) \quad ; \quad \mu_{P_1}^j(x(j)) = \exp(-x_j(0.5j))$$

$$\mu_{P_2}^j(x(j)) = \frac{1}{1 + \exp(5(x(j) + 2)))} \quad ; \quad \mu_{P_3}^j(x(j)) = \frac{1}{1 + \exp(-5(x(j) - 2)))}$$

$$\mu_{P_4}^j(x(j)) = \exp(-x(j) + 0.5j) \quad ; \quad \mu_{P_5}^j(x(j)) = \exp(-x(j) - 1.5j) \quad (12)$$

Control strategy is based on controlling the $X_1$ to follow the sinusoidal wave. So any differential from the reference amount appears as error that will be injected to the AFLC as an input. The other input is the initial value of parameter $\Theta$. With the adaption low of $\theta^0 = \gamma \Theta^T P_{e, 0} \xi(x)$ the parameter is adjusted in each iteration. The output control of $U$ as $U = \Theta^T \tilde{x}(x)$ is applied to the chaos system in each cycle. System equipping with $u$ control is shown in (13)

$$X_1 = X_2 \quad X_2 = -X_1^3 - 1.5 X_2 + \cos t + U \quad (13)$$

The amount of crisp control value depends on its previous content. In the case of great or few variations in state $x_i$ in the chaotic system, the AFLC reflection will be entirely nonlinear according to the fact that the AFLC compensates the control system in conjunction to a nonlinear conversion.

IV. MATLAB SIMULATION

Plant model and controller are simulated in Matlab simulink. Simulation results are presented to demonstrate the feasibility of the AFLC method through its effectiveness. Chaotic plant model of the autonomous ordinary differential equations ODEs evolving “$u$” control (13) is simulated in Matlab as fig. 4.

In all of the following simulations, the initial conditions are chosen as $[X_1(0), X_2(0)] = [2, 2]$ and $[\theta_1(0), \theta_2(0)] = [1, 1]$.
Six membership functions for each of $x_1$ and $x_2$ between interval of 0-5 and 0-300 seconds are shown in Fig. 6. The input membership functions in input stage, are introduced to substitute inputs with the crisp values to fuzzified variables and in the output stage the output membership functions contribute to make a crisp output trough the defuzzification part. In AFLC all the membership functions in inputs and outputs are varying and adapting to minimize the error using the adaption low.

Variation takes place during the simulation time (0-300) seconds. For more details the interval 0-5 is shown too in Fig. 6. The below figures indicate the $X_1$ and $X_2$ in interval 0-300 using AFLC. As it is declare the final destination for both $X_1$ and $X_2$ trajectory is around (-1 to 1).

CONCLUSION

AFLC presents the empowered methodology employing the adaptive fuzzy logic controller to fully control the Duffing’s chaotic system following the elliptical trajectory. It also surmounts the limitations and problems in chaotic system like nonlinearity and rapid variation in the state space. It is more advanced in compare with the “disturbance-based technique”, “OGY” and Pyragas method. It also works much better than convenient FLC controller. AFLC could be expanding to use in many chaotic systems which propones high performance result and robustness advantages that will enthral the designers in finding solutions in case of such nonlinear chaotic phenomena.

REFERENCES

Robust Adaptive Fuzzy Logic Approach In Controlling the Duffing’s Chaotic System


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