## **OPTIMIZATION TECHNIQUE (PSO & APSO) TO CRACK BIDDING PROBLEM FOR PARTICIPANT IN ELECTRICITY MARKET**

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**Abstract-** In an open competitive electricity market generators (supplier) and large consumer (buyer) need a suitable bidding model for enhancing their profits. Therefore, each generators (supplier) and large consumer (buyer) will bid strategically for the selection of bidding coefficients to check out the rivals bidding strategy. In this paper, bidding strategy problem is treated as an optimization problem and solved using Particle Swarm Optimization (PSO) and Adaptive Particle Swarm Optimization (APSO). First we have initialized The system with a population of random solutions and searches for optimal result in problem space by updating generation. In PSO, the potential solutions, known as particles, fly through the problem space in all direction by following the current optimum solution (particle). APSO is proposed to improve the performance of PSO (i.e. weight update technique is different in APSO and weight vary according to performance of particle). A numerical problem with six Generator (suppliers) and two large consumers (buyer) is used to describe the essential features of the proposed method. The results indicate that the APSO is better than PSO with respect to total profit.

**Keywords**- Prticle Swarm Optimization (PSO), Adaptive Prticle Swarm Optimization (APSO), Market Clearing Price (MCP), Bidding Strategy, Competitive Electricity Market.

## I. INTRODUCTION

The power market has change significantly over the past few years. This is mainly due to three factoremergence of competitive bidding, growth of bilateral trading and introduction of power exchange. After the Electricity Act, 2003 many new reform have been done in power sector like allowing private companies in generating sector. By this reform, there is a need of platform in which all transaction can be done between buyers and supplier.

Around the world, such transaction are carried out by power exchange and many countries have already their power exchange. An ample amount of literature have been publish about bidding in electricity market and such power exchange. Before deregulation a traditional monopoly structure was exit in the power sector market. After deregulation process the Large consumer (buyers) and generators (suppliers) starts to interact regarding power transaction and maintain system security through system operator. Competitive electricity market consists of several Generating Companies, Transmission Companies and Distribution Companies along with the system operators.

There are different type of bid use by different power exchange and that are bound by different set of bidding and auction protocols. Bidding problem start here as generating companies know their production costs, technical parameter and behaviour of other participant and electricity market behaviour, so the immediate probem for the generators (suppliers) is that, how to get best optimal bid. This problem is considered as a strategic bidding problem in electricity market. To solve bidding problem we have broadly three approach.

• The first one is based on estimations of the MCP.

• The second is based on estimations of bidding behaviour of the rival participants.

• The third is based on game theory.

Among three approach the first one is easy and that is use by most of power exchange around the world. Earlier the bidding problem was solved by many optimization technique. All technique are able to solve the bidding problem however they have some limitation in choice of their parameter and other issues (i.e. execution time, number of iteration etc.).

The main work of this paper is the bidding problem is solved by new optimization technique known as Particle Swarm Optimization (PSO) and Adaptive Particle Swarm Optimization (APSO). The supremacy of Adaptive Particle Swarm Optimization (APSO) than other techniques is discuss below and that is supported by simulation result also.

## **II. BIDDING SCENARIO IN INDIA**

Power exchanges in India was commence in 2008. There was a need for a market place in India, where large consumer (buyers) and generators (sellers) could meet and buy or sell power with genuine price discovery. The motivation for establishing such market place in India comes from the Electricity Act, 2003, which is the first act to introduced the concept of non-discriminatory open access for power through rules and regulation for promoting competition in the electricity market. As the major step taken by the Electricity Act, 2003 the country's power markets have been witnessing significant innovation. Further efforts are positive regulatory that create a competitive market and supported by the efforts of market operators to introduce new products and solutions that benefit consumers, suppliers and the power sector as a whole. Before the functioning of power exchanges in India, an alternatives method was used for purchasing short-term power that consist the unscheduled interchange (UI) market (where prices were volatile) and over the-counter (OTC) trading mechanisms (which typically have high transaction costs). Only the OTC mechanisms continue to serve an important function, earlier consumers wanted a platform that allowed them to enter standardized contracts, take care of counterparty risks, and provided fixed acceptable future electricity price signals. The customer demand for such contracts led to the evolution of power exchanges in India.

The IEX provides a platform for trading power in two type of market first is the day-ahead market (DAM) and second is the term-ahead market (TAM). IEX also start operation in near future for the renewable energy segment.

#### **III. PROBLEM FORMULATION**

Lets take an example of a power market in which 'm' no. of generators are participating in bidding and it consist an inter connected network controlled by an Independent System Operator (ISO), a Power Exchange (PX), total consumer load which is not part of bidding in electricity market but it is depend on the price of electricity, and there are 'n' no. of large consumers (buyers) who participate in demand-side bidding.

PX define the range in which generators (supplier) and large consumer (buyers) will bid. The bidding is done as, the generators (supplier) and large consumer (buyers) will bid in a linear non decreasing supply function and non increasing demand function. Linear supply curve for bidding of a generators is denoted as

Gi(Pi)=ai+biPi where i=1..m

Linear demand curve for bidding of a large consumer is denoted as

Wj(L)=cj-djLj where j=1...n

In above equation Pi is the active power output of generators, ai and bi are the supply bidding coefficients of the ith supplier. Lj is the active power load of large consumer, cj and dj are the demand bidding coefficients of the jth large consumer. Supply and demand bidding coefficient (ai, bi, cj system

parameter are ignored, then enhancing profits leads to a uniform market clearing price for all participant. Now, when only the load flow limit and generators (suppliers) output limit and large consumers (buyers) demand limit are taken into account then PX will determines a set of generators (supplier) outputs P = (P1,P2,P3...Pm)T and a set of large consumers (buyers) demands L = (L1, L2, L3, ..., Ln)T by solving equations (1) to (5) shown below. and dj) are positive. The function of PX is to calculate the Pi/Lj which must be within the limit of system operating parameter and that must be meets security parameter of the system. This leads to maximum profit and social welfare. It is observed from many electricity market that when the generators (suppliers) and large consumers (buyers) bid in the linear supply function and linear demand functions and the

ai + biPi = R i = 1, 2...m (1)

$$cj - djLj = R \quad j = 1, 2 \dots n$$
 (2)

$$\sum_{i=1}^{m} P_{i} = Q(R) + \sum_{j=1}^{n} L_{j}$$
(3)

 $Pmin, i \le Pi \le Pmax, i \ i = 1, 2 \dots m \ (4)$  $Lmin, j \le Lj \le Lmax, j \ j = 1, 2 \dots n \ (5)$ 

Where R is the uniform market clearing price of electricity to be determined and Q(R) is the aggregate pool load forecast by PX and this information made public for all participants and is considered as it will be elastic to the price of electricity in the market. Here Pmax and Pmin are the generators (suppliers) output limits of the ith supplier, and Lmax and Lmin are the demand limits of the jth large consumer (buyers).

The mathematical formula for Q(R) is define below and equations (1) to (3) can be solved directly. Assume that the aggregate pool load Q(R) follow the linear form shown below.

$$Q(R) = Q_0 - KR \tag{6}$$

In the above equation Qo is a constant number and K is a positive coefficient that show price elasticity of the aggregate demand (i.e. depend on the price of electricity). If pool demand does not depend on price of electricity, then K=0.

The condition given in equation (4) and (5) are ignored, the solutions to equations (1) to (3) are given below.

$$R = \frac{Qo + \sum_{i=1}^{m} \frac{ai}{bi} + \sum_{j=1}^{n} \frac{cj}{dj}}{K + \sum_{i=1}^{m} \frac{1}{bi} + \sum_{j=1}^{n} \frac{1}{dj}}$$
(7)

$$Pi = (R-ai)/bi$$
  $i=1,2...m$  (8)

$$Lj=(cj-R)/dj$$
  $j=1,2...n$  (9)

When the equation (8) and (9) violates generators output limit (4) and consumer demand limits (5) then it must be rearrange according to given limits. Let the ith generator follow a cost function define as

$$Ci(Pi) = eiPi + fiPi$$

where ei and fi are cost function coefficients. For enhancing the profit of generator a bidding strategy is adopted, which is shown below.

This equation is subjected to equation (1) to (5). This equation is modified by putting the value of cost function as.

$$F(ai, bi) = (ai - bi)Pi + (bi - fi)Pi^2$$
(10)

The main objective to calculate ai and bi so as to enhancing the profit F(ai,bi) with the help of equation (1) to (5). Ci(Pi) is the production cost function of the ith generators (suppliers). Now the ith large consumer (buyers) follow a revenue function define as Bj(Lj) = gjLj - hjLj, here gj and hj are the demand function coefficients. For enhancing the profit of generator a bidding strategy is adopted, which is shown below.

#### Maximize B(cj,dj) = Bj(Lj) - RLj

This equation is subjected to equation (1) to (5). This equation is modified by putting the value of revenue function as.

$$F(cj,dj) = (gj - cj)Lj - (hj - dj)Lj^{2}$$
(11)

In the competitive electricity market sealed based auction method is adopted. In this method bidding data for the next time is confidential. So this is the problem for generators (suppliers) and large consumers (buyers) to solve the equation (10) and (11). But the past bidding data is available, and this data made public for all participant, so they can make use of this information for bidding problem. But the next problem for each participant is to forecast the bidding coefficients of rivals (large consumers). Let assume that the ith generators (supplier' s) has forecast that rival' s jth (j  $\neq$  i) bidding coefficients follow a joint normal distribution function with the following probability density function (PDF). The probability density function (PDF) is define below.

$$PDFi (aj, bj) = \frac{1}{2 \times \pi \times \sigma_j^{(a)} \times \sigma_j^{(b)} \times \sqrt{1 - \rho j^2}} \times \exp\left[-\frac{1}{2 \times (1 - \rho j^2)} \left[ \left(\frac{aj - \mu_j^{(a)}}{\sigma_j^{(a)}}\right)^2 - \frac{2 \times \rho j \times \left(aj - \mu_j^{(a)}\right) \times \left(bj - \mu_j^{(b)}\right)}{\sigma_j^{(a)} \times \sigma_j^{(b)}} + \left(\frac{bj - \mu_j^{(b)}}{\sigma_j^{(b)}}\right)^2 \right] \right]$$
(12)

Where  $\rho j = \text{correlation coefficient between ai and bi..}$  $\mu j(a), \mu j(b) = \text{mean value (Parameter of joint normal distribution).}$ 

 $\sigma j(a)$ ,  $\sigma j(b)$  = standard deviation (Parameter of joint normal distribution).

Same probability density function (PDF) can be written for large consumers also. Which will be used for finding the bidding coefficient of large consumers. Now with probability density function (PDF) and equation (10) & (11) subjected to condition given in equation (1) to (5) becomes a stochastic optimization problem. That is solve with the help of optimization technique. In this paper I am using PSO & APSO for solving this stochastic problem.

# IV. PARTICLE SWARM OPTIMIZATION (PSO)

The PSO technique is an un-supervise optimization technique that is based on social interaction such as bird flocking and fish school. This technique is suitable for any nonlinear or random optimization problem. It was first discovered in 1995 by a social psychologist James Kennedy and an electrical engineer Russell Eberhart. The basic concept of PSO is that, the optimized result obtained is called as particles and the particle try to fly through the problem space in N dimension by tracking the best optimal result so far of the particles. It has some basic similarity with the available computation techniques in the market such as initialization. In PSO initialization is done as, first a mass of random solution is taken and then search for optimal solution by updating the particle weight. In PSO each particle is considered just as a point in a N-dimensional problem space. This point adapt its flying according to its own flying experience as well as the flying experience of other neighboring particles in problem space. Equation (13) written below is used for updating the velocity, at each iteration a modified velocity is obtained for each particle based on its previous velocity (Vr k), the particle's location at which the best fitness has been calculated (Pbest r) so far, and the best particle among the neighbors (Gbest r) at which the best fitness has been calculated so far.

The learning factors C1 and C2 are the acceleration constants that change the velocity of a particle towards (Pbest k) and (Gbest k), and rand1, rand2 are uniformly distributed random numbers in [0, 1]. Each particle's position is updated using equation (14) in the solution space. The velocity is update by using equation (15).

$$V_r^{k+1} = W^k \times V_r^k + C1 \times rand1 \times (P_{best}^k - X_r^k)$$
  
+ C2 \times rand2 \times (G\_{best}^k - X\_r^k) (13)

$$X_r^{k+1} = X_r^k + V_r^{k+1} \tag{14}$$

$$W^{k} = W_{max} - \frac{W_{max} - W_{min}}{K_{max}} \times K$$
(15)

Where Vr k: It is the velocity of particle r at iteration k Wk: Weight at kth iteration.

C1, C2: Acceleration factor.

rand1, rand2: uniformly distributed random number between 0 and 1.

 $Xr \; k: current \; position \; of \; particle \; r \; at \; iteration \; k.$ 

Pbest k : Best fitness of particle at kth iteration.

Gbest k: Best fitness of group at kthiteration.

Xr k+1: New position of particle.

Wmax: Maximum weight.

Wmin: Minimum weight.

Kmax: Maximum Iteration.

K: Iteration.

The velocity update equation (13) have three term and all have their own significance in updating velocity. The significance of first term which contain inertia is that, it will continue to fly the particle in the same direction until it get first result. Therefore we can say first term is responsible for exploring new areas in problem space. If first term is not part of velocity update equation then the velocity of the particle is only calculated by current position and best position in history. So the first term is very essential to get optimal solution. The second term representing memory and third term representing cooperation. All three term together try to converge the particles to their (Pbest k) and (Gbest k) in the search procedure.

A. Adaptive Particle Swarm Optimization (APSO) This is the further modification in the conventional PSO. As both are optimization technique and learn from the surrounding particles, but APSO [13] is little advance in the search for optimal solution. In conventional PSO algorithm, non optimal particles have a tendency to shift near the location of Gbest. Therefore, the global optimal particle must explore new areas and update the Gbest to give momentum to the search of other particles. In this optimization technique an adaptive PSO algorithm is proposed to improve its performance. In this approach Different particles are allocated with different tasks. As in case of conventional PSO, we define weight at starting with maximum weight and minimum weight. But in this technique we can vary the weight according to the performance or task of particles. The particles with better performance have larger inertia weight, which allocate the task of searching better area. The particles which have poor performance are assign by a smaller inertia, allowing them to quickly converge to a better area for detailed search. By the variation in weight a large inertia weight is responsible for a global search while a small inertia weigh responsible for a local search. The particles are arrange in order of their individual optimal location from excellent to worst. The weight update formula for APSO is describe below. In this technique acceleration constant also update after every iteration, the update formula is describe below.

 $Wi=Wmin+(Wmax-Wmin)\times((m-i))/((m-1)) (16)$ 

$$Ci1=Ci2=(Wi+1+2\times\sqrt{Wi})/2$$
 (17)

Where m is define in above equation as population size, inertia weigh Wi is adjusted according to the above equation. By this technique both global and local search can be done in each iteration step. In APSO velocity and position will be update same as in case of PSO, but weight and acceleration factor will be update according to equation (16) & (17).

1. APSO algorithm for obtaining optimal bidding coefficients (bi/dj).

Step1. Initialization of the particles

(a) Initialize randomly the population of bi solutions in a matrix form. where bi is the bidding parameter of the ith generators (suppliers) to be optimized.

(b) Read input data  $\mu$  ,  $\sigma$  ,  $\rho,ai$  and maximum iterations.

where  $\mu$ = mean ,  $\sigma$ =standard deviation,  $\rho$  =correlation coefficient of probability density function (PDF), ai= cost coefficient of generators.

Step2. Calculate the fitness evaluation function for each individual bi, by the equation (12). Here probability density function (PDF) is Fitness evaluation.

Step3. Now each Pbest values are compared with the other Pbest values in the population. The best value among the Pbest is replace as Gbest.

Step4. Now update the velocity V by velocity update equation (13) of each individual bi

Step5. Now update the position by position update equation (14) of each individual bi

Step6. Update the weight Wi for each individual bi is according to the weight update equation (16).

Step7. Update the acceleration factor Ci1,Ci2 for each individual bi is according to the acceleration factor update equation (17).

Step8. Repeat from steps 3-7 until iteration reaches their maximum count. Return the best optimal value of bi. At final iteration as a global fitness. Using bi values, calculate MCP from equation (7).

A similar algorithm is applied to find the optimal values of dj.

2. APSO algorithm for profit calculation of Buyers and Suppliers

Step1. Initialization of the particles

(a) initialize randomly population of power Pi solutions where Pi is the power of the ith supplier.

(b). Read input data of Generators (i.e. cost coefficients, Pmax, Pmin), demand (Q0) and maximum number of iterations.

Step2. Calculate the Fitness evaluation function by using equation (10) and (11).

Step3. Now each Pbest values are compared with the other Pbest values in the population. The best evaluation value among the Pbest is replace as Gbest.

Step4. Now update the velocity V by velocity update equation (13) of each individual Pi.

Step5. Now update the position by position update equation (14) of each individual Pi.

Step6. Update the weight Wi for each individual bi is according to the weight update equation (16).

Step7. Update the acceleration factor Ci1,Ci2 for each individual bi is according to the acceleration factor update equation (17).

Step8. Repeat from steps 3-7 until iteration reaches their maximum count. Return the best fitness value of power within the given limit and maximum profit.

At final iteration both value considered as global solution. APSO gives the more global solution than PSO, as at every iteration the weight and acceleration factor is updating according to equation (16) & (17). It will search more optimal solution as at starting some of the particle have different weight and after first iteration weight is assigned according to performance. So search start from poor performance and end at better performance.

#### V. RESULT AND DISCUSSION

As we have consider there are six generators (suppliers) and two large consumer (buyers). The data

for Generators and large consumer is given in the Table 1. The value of some mathematical coefficient used in bidding problem is also given as Qo(a constant number) is 300 and K (coefficient denoting the price elasticity of the total demand) is 5. In this paper, the other parameters related to PSO/APSO are used after fine tuning are as, Population size: 50, accelerating factors (for PSO only), C1=C2=2.0, inertia weight W (Wmax,Wmin): 1.0 to 0.5, Maximum number of iterations:

150. Simulations are carried on 1.80GHz, Intel(R) core(TM) i5-3337U Processor, 6GB RAM and MATLAB R2010 version is used.

Table1.	Generator	and	Large	Consumer	Data	Given
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Generator No.	e	f	Pmin(MW)	Pmax(MW)
1	6.00	0.01125	40	160
2	5.25	0.0525	30	130
3	3.00	0.1375	20	90
4	9.75	0.02532	20	120
5	9.00	0.075	20	100
6	9.00	0.075	20	100
Large consumer	g	h	Lmin(MW)	Lmax(MW)
1	30.00	0.04	0	200
2	25.00	0.03	0	150

e: Cost function coefficient of ith generator.

f: Cost function coefficient of ith generator.

g: Demand function coefficient of ith generator.

h : Demand function coefficient of ith generator.

Pmin(MW), Pmax(MW) : Generator limit.

Lmin(MW), Lmax(MW) : Demand limit.

The data given in the table above is used to solve the bidding probem and the bidding result for APSO and PSO is shown in the table below.

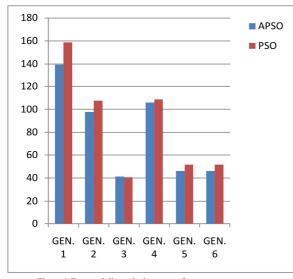
Table 2. Bidding coefficient for Generators and Large Consumers

	APSO	PSO
Generators no.	bi	bi
1	0.081	0.057
2	0.077	0.069
3	0.259	0.245
4	0.057	0.053
5	0.165	0.104
6	0.165	0.125
Large consumer no.	dj	dj
1	0.083	0.082
2	0.056	0.051

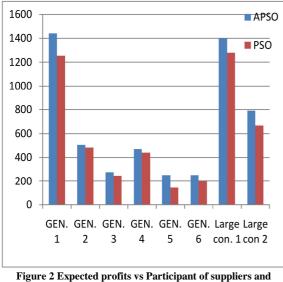
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	APSO		PSO	
Generator no.	P (MW)	Profit (\$)	P (MW)	Profit (\$)
1	139.25	1440.3	158.66	1253.53
2	97.65	505.48	107.53	482.50
3	41.31	277.03	40.61	247.49
4	105.94	470.00	108.71	438.87
5	46.11	251.64	51.60	144.04
6	46.11	251.64	51.60	198.62
Large consumer no.	L (MW)	Profit (\$)	L (MW)	Profit (\$)
1	164.13	1400.9	157.97	1278.60
2	143.86	792.34	140.53	667.21
МСР	17.62	I	15.89	
Total Profit	7883.12		4710.88	

Optimization Technique (PSO & APSO) To Crack Bidding Problem For Participant In Electricity Market Table 3. MCP (\$/MWh) and Profit (\$) of Generators and Large Consumers







consumers

## CONCLUSION

APSO Technique show much better result as compare

to PSO in following respect. The overall profit of both buyer and supplier has increased using APSO. So supplier has more money to invest in power generation and buyer has more money to purchase more electricity. As we see in APSO, the generators are dispatching power with in the limit or we can say below its maximum rating, so it will leads to increase the life of generator. Third we see that MCP is increased with APSO, so power Exchange will also benefited. It will leads to better management of power exchange.

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